NAG Toolbox for MATLAB

f07vv

1 Purpose

f07vv returns error bounds for the solution of a complex triangular band system of linear equations with multiple right-hand sides, $AX = B$, $A^{\mathrm{T}}X = B$ or $A^{\mathrm{H}}X = B$.

2 Syntax

```
[ferr, berr, info] = f07vv(uplo, trans, diag, kd, ab, b, x, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07vv returns the backward errors and estimated bounds on the forward errors for the solution of a complex triangular band system of linear equations with multiple right-hand sides $AX = B$, $A^{\mathrm{T}}X = B$ or $A^{\mathrm{H}}X = B$. The function handles each right-hand side vector (stored as a column of the matrix B) independently, so we describe the function of f07vv in terms of a single right-hand side b and solution x.

Given a computed solution x, the function computes the *component-wise backward error* β. This is the size of the smallest relative perturbation in each element of A and b such that x is the exact solution of a perturbed system

$$(A + \delta A)x = b + \delta b$$
$$|\delta a_{ij}| \leq \beta |a_{ij}| \quad \text{and} \quad |\delta b_i| \leq \beta |b_i|.$$

Then the function estimates a bound for the *component-wise forward error* in the computed solution, defined by:

$$\max_i |x_i - \hat{x}_i| / \max_i |x_i|$$

where $\hat{x}$ is the true solution.

For details of the method, see the F07 Chapter Introduction.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates whether A is upper or lower triangular.

uplo = 'U'

A is upper triangular.

uplo = 'L'

A is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **trans – string**

Indicates the form of the equations

trans = 'N'

The equations are of the form $AX = B$.

trans = 'T'

The equations are of the form $A^{\mathrm{T}}X = B$.

trans = 'C'

The equations are of the form $A^{\mathrm{H}}X = B$.

Constraint: **trans** = 'N', 'T' or 'C'.

3: **diag – string**

Indicates whether A is a nonunit or unit triangular matrix.

diag = 'N'

A is a nonunit triangular matrix.

diag = 'U'

A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = 'N' or 'U'.

4: **kd – int32 scalar**

k_d, the number of superdiagonals of the matrix A if **uplo** = 'U', or the number of subdiagonals if **uplo** = 'L'.

Constraint: **kd** ≥ 0.

5: **ab**(**ldab,**$*$) **– complex array**

The first dimension of the array **ab** must be at least **kd** $+ 1$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n triangular band matrix A.

The matrix is stored in rows 1 to $k_d + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(k_d + 1 + i - j, j)$ for $\max(1j - k_d) \leq i \leq j$;
if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in $\mathbf{ab}(1 + i - j, j)$ for $j \leq i \leq \min(nj + k_d)$.

If **diag** = 'U', the diagonal elements of A are assumed to be 1, and are not referenced.

6: **b**(**ldb,**$*$) **– complex array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r right-hand side matrix B.

7: **x**(**ldx,**$*$) **– complex array**

The first dimension of the array **x** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r solution matrix X, as returned by f07vs.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **ab**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

Default: The second dimension of the array **b** The second dimension of the array **x**.

r, the number of right-hand sides.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

ldab, ldb, ldx, work, rwork

5.4 Output Parameters

1: **ferr**$(*)$ **– double array**

Note: the dimension of the array **ferr** must be at least $\max(1, \mathbf{nrhs\_p})$.

ferr(j) contains an estimated error bound for the jth solution vector, that is, the jth column of X, for $j = 1, 2, \ldots, r$.

2: **berr**$(*)$ **– double array**

Note: the dimension of the array **berr** must be at least $\max(1, \mathbf{nrhs\_p})$.

berr(j) contains the component-wise backward error bound β for the jth solution vector, that is, the jth column of X, for $j = 1, 2, \ldots, r$.

3: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **trans**, 3: **diag**, 4: **n**, 5: **kd**, 6: **nrhs_p**, 7: **ab**, 8: **ldab**, 9: **b**, 10: **ldb**, 11: **x**, 12: **ldx**, 13: **ferr**, 14: **berr**, 15: **work**, 16: **rwork**, 17: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The bounds returned in **ferr** are not rigorous, because they are estimated, not computed exactly; but in practice they almost always overestimate the actual error.

8 Further Comments

A call to f07vv, for each right-hand side, involves solving a number of systems of linear equations of the form $Ax = b$ or $A^{\mathrm{H}}x = b$; the number is usually 5 and never more than 11. Each solution involves approximately $8nk$ real floating-point operations (assuming $n \gg k$).

The real analogue of this function is f07vh.

9 Example

```
uplo = 'L';
trans = 'N';
diag = 'N';
kd = int32(2);
ab = [complex(-1.94, +4.43), complex(4.12, -4.27), complex(0.43, -2.66),
complex(0.44, +0.1);
      complex(-3.39, +3.44), complex(-1.84, +5.53), complex(1.74, -0.04),
complex(0, 0);
          complex(1.62,  +3.68),  complex(-2.77,  -1.93),  complex(0,  0),
complex(0, 0)];
b = [complex(-8.86, -3.88), complex(-24.09, -5.27);
     complex(-15.57, -23.41), complex(-57.97, +8.14);
     complex(-7.63, +22.78), complex(19.09, -29.51);
     complex(-14.74, -2.4), complex(19.17, +21.33)];
[x, info] = f07vs(uplo, trans, diag, kd, ab, b);
[ferr, berr, info] = f07vv(uplo, trans, diag, kd, ab, b, x)
```

```
ferr =
   1.0e-13 *
    0.1803
    0.2199
berr =
   1.0e-16 *
    0.0904
    0.6571
info =
           0
```